

# The inflaton as dark matter

S. Barshay, G. Kreyerhoff

III. Physikalisches Institut A, RWTH Aachen, D-52056 Aachen, Germany

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**Abstract.** Within the framework of an explicit dynamical model, in which we calculate the radiatively-corrected, tree-level potential that sets up inflation, we show that the inflaton can be a significant part of dark matter today. We exhibit potentials with both a maximum and a minimum. Using the calculated position of the potential minimum, and an estimate for fluctuations of the inflaton field in the early universe, we calculate a contribution to the matter energy density of  $(1 - 2) \times 10^{-47}$  GeV<sup>4</sup> in the present universe, from cold inflatons with mass of about  $6 \times 10^9$  GeV. We show that the inflaton might decay in a specific way, and we calculate a possible lifetime that is several orders of magnitude greater than the present age of the universe. Inflaton decay is related to an interaction which, together with a spontaneous breakdown of CP invariance at a cosmological energy scale, can give rise to a neutrino-antineutrino asymmetry just prior to the time of electroweak symmetry breaking.

## 1 Introduction

The purpose of this paper is to present the results of calculations which support two new ideas concerning matter in the universe:

- (1) A large part of dark matter in the present universe can be composed of massive, cold inflatons, the scalar quantum of the field whose vacuum energy initiated a period of inflation [1–3].
- (2) There is a definite possibility that the inflatons are not absolutely stable; they can decay in a specific way, with a possible lifetime estimated here to be several orders of magnitude greater than the present age of the universe.

The above results are based upon detailed calculations within a specific dynamical model [4]. These calculations show, first, that radiative corrections to the tree-level potential for the scalar field,  $\phi$ , can set up inflation [4]. The radiative corrections are calculated via the renormalization-group equations, using the potential for  $\phi$  and quantum interactions for definite additional fields: a massive, neutral lepton  $L$ , and a massive, spin-zero boson  $b$ . (The primordial fields are electrically neutral.) The calculated radiatively-corrected potential has a maximum at an energy scale near to the Planck scale  $M_P = 1.2 \times 10^{19}$  GeV, and a minimum below this scale, at  $\phi = \phi_c$ . We consider that inflation occurs while the scalar field rolls down from this maximum toward the minimum. (Much can occur at the maximum.)

In obtaining the above results (1,2) we shall use explicit, calculated examples of the radiatively-corrected potential,  $V_c(\phi) = \lambda(\phi)\phi^4$ ; these potentials are shown in

Correspondence to: georg@physik.rwth-aachen.de

Fig. 1 and Fig. 2. We briefly recapitulate the method of calculation [4]. The potential and quantum interactions are given by<sup>1</sup>.

$$V(\phi, b) = \lambda\phi^4 + \lambda_b\phi^2b^2 + \tilde{\lambda}_b b^4 + g\bar{\psi}_L\psi_L\phi + g_b\bar{\psi}_L\psi_L b \quad (1)$$

All coupling parameters are dimensionless. The equations giving the development of the couplings with the energy scale  $\phi$  are

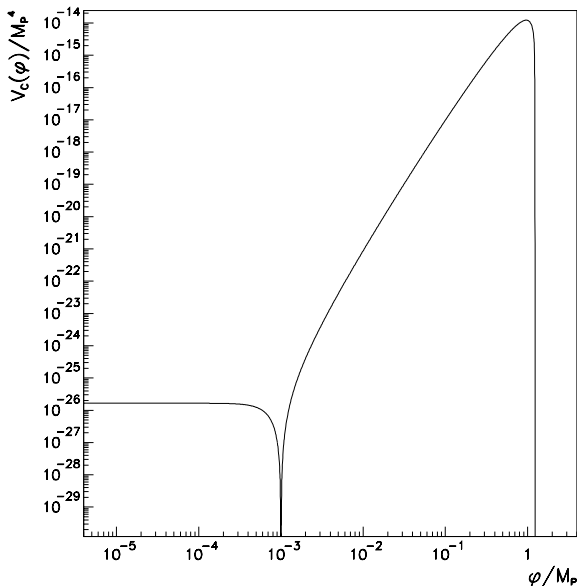
$$\begin{aligned} 16\pi^2 \frac{d\lambda(\phi)}{dt} &= 72\lambda^2 + 2\lambda_b^2 - 2g^4 = \beta_\lambda(\phi) \\ 16\pi^2 \frac{d\lambda_b(\phi)}{dt} &= \lambda_b(24\lambda + 16\lambda_b + 24\tilde{\lambda}_b) - 4g^2g_b^2 \\ &= \beta_{\lambda_b}(\phi) \quad (2) \\ 16\pi^2 \frac{d\tilde{\lambda}_b(\phi)}{dt} &= 72\tilde{\lambda}_b^2 + 2\lambda_b^2 - 2g_b^4 = \beta_{\tilde{\lambda}_b}(\phi) \\ 16\pi^2 \frac{dg(\phi)}{dt} &= 5g(g^2 + g_b^2) = \beta_g(\phi) \\ 16\pi^2 \frac{dg_b(\phi)}{dt} &= 5g_b(g_b^2 + g^2) = \beta_{g_b}(\phi) \end{aligned}$$

The extremum conditions [4,5] are imposed at the scale  $\phi = \phi_i \lesssim M_P$ , at which the computations are begun,

$$\begin{aligned} \beta_\lambda(\phi_i) &= 0 \\ \beta_{\lambda_b}(\phi_i) &= 0 \quad (3) \end{aligned}$$

For given values of  $\lambda(\phi_i)$  and  $\lambda_b(\phi_i)$  (with <sup>2</sup>  $\tilde{\lambda}_b(\phi_i) =$

<sup>1</sup> For pseudoscalar  $b$ ,  $\bar{\psi}_L\psi_L b \rightarrow i\bar{\psi}_L\gamma_5\psi_L b$ . The results in Figs. 1,2 are unchanged



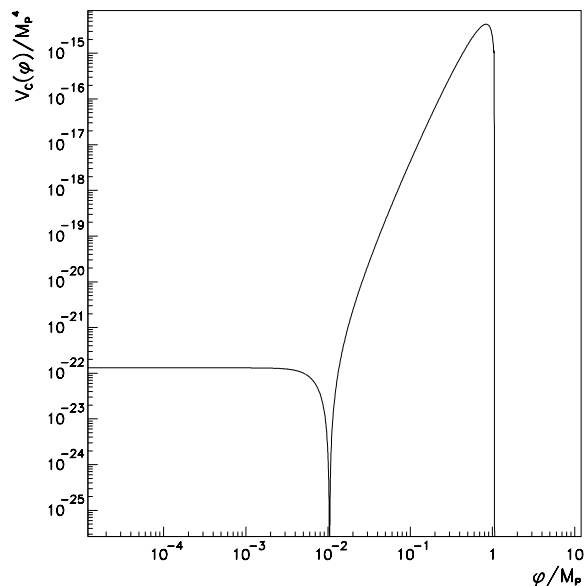
**Fig. 1.** An example of the radiatively-corrected, tree-level potential  $V_c(\phi)$  which sets up inflation, as calculated by solving (2,3) in the text. The maximum is at  $\phi = \phi_m \cong M_P \cong 10^{19}$  GeV, and the minimum is at  $\phi = \phi_c \cong 10^{-3}M_P \cong 10^{16}$  GeV. The inflaton mass is  $m_\phi \cong 5 \times 10^{-10}M_P$ . (Addition of a constant renormalizes the calculated curve so that  $V_c(\phi_c) = 0$ .)

$\lambda_b(\phi_i)$ ), these conditions *determine*  $g(\phi_i)$  and  $g_b(\phi_i)$ . The chosen values for  $\lambda(\phi_i)$ ,  $\lambda_b(\phi_i)$  were determined [4], in order of magnitude, by conditions during the period of inflation, i. e. “slow roll-down” and “sufficient expansion” [6]. In this paper, we deal with physical processes which occur at times after  $\phi$  has fallen through the minimum of the potential. The renormalization group equations (2) are solved numerically. The potential shown in Fig. 1 has  $\phi_i = (4 \times 10^{-2})M_P$ ,  $\lambda(\phi_i) = 10^{-13}$  and  $\lambda_b(\phi_i) = 10^{-4}$ . The calculated position of the maximum is  $\phi = \phi_m = M_P$ , and the calculated position of the minimum is  $\phi = \phi_c = 10^{-3}M_P$ . The calculated initial values for the coupling parameters are  $g(\phi_i) = 10^{-2}$ , and  $g_b(\phi_i) = 3.2 \times 10^{-2}$ . The potential shown in Fig. 2 has  $\phi_i = 10^{-1}M_P$ ,  $\lambda(\phi_i) = 4.3 \times 10^{-14}$ , and  $\lambda_b(\phi_i) = 10^{-4}$ . The maximum is at  $\phi_m = M_P$ ; the minimum is at  $\phi_c = 10^{-2}M_P$ . The initial couplings are  $g(\phi_i) = 10^{-2}$  and  $g_b(\phi_i) = 3.2 \times 10^{-2}$ . In addition, the mass of the inflaton is calculated from  $d^2V_c(\phi)/d\phi^2|_{\phi=\phi_c} = m_\phi^2$ ; this is  $(6 \times 10^9 \text{ GeV})^2$  for Fig. 1 and  $(5 \times 10^{10} \text{ GeV})^2$  for Fig. 2.

## 2 The matter energy density of inflatons

The discussion which follows on the basis of these potentials is largely phenomenological, as must be the case for a discussion of interaction and decay processes which involve hypothetical massive particles in the early universe.

<sup>2</sup> Smaller  $\tilde{\lambda}_b (\neq \lambda_b)$  move the calculated maximum and minimum further away from  $\phi_i$ .  $\tilde{\lambda}_b$  can be made much smaller, and the effect compensated by increasing  $\lambda_b$  slightly



**Fig. 2.** A second example of the radiatively-corrected, tree-level potential. The maximum is at  $\phi = \phi_m \cong M_P$ , and the minimum is at  $\phi = \phi_c \cong 10^{-2}M_P$ . The inflaton mass is  $m_\phi \cong 4 \times 10^{-9}M_P$

The numerical work is meant to be indicative of a general picture for the development in time of the matter (and radiation) energy densities. Consider first the (squared) fluctuations  $(\delta\phi)^2$  of the inflaton field in the vicinity of the minimum. We consider that there is a matter energy density  $\rho_M^\phi$ , given by the inflaton (squared) mass,

$$\rho_M^\phi(t_c) \cong \frac{1}{2}m_\phi^2(\delta\phi)^2 \quad (4)$$

The fluctuations can develop semiclassically [7] to a maximum value [8,9], which we parametrize in two alternative forms<sup>3</sup>

$$(\delta\phi)^2 \cong \epsilon\phi_c^2 \quad (5a)$$

$$(\delta\phi)^2 \cong \epsilon\phi_c\phi_i \quad (5b)$$

where  $\epsilon$  is a small parameter. As we shall discuss below, independent estimates [8,9] give values which lie in the interval

$$10^{-7} \leq \epsilon \leq 10^{-5} \quad (6)$$

Note that  $\epsilon$  is approximately given by  $\sqrt{\lambda}$ . We assume that the squared mass  $m_\phi^2$ , and the maximal squared fluctuations  $(\delta\phi)^2$ , together provide a measure of the initial matter energy density. This at about a time  $t_c \cong$

<sup>3</sup> The form (5a) for “scaled” fluctuations is well-known in high energy particle collisions. Under the name [10,11] “KNO scaling”, it provides an approximate representation, in a certain energy interval, for the fluctuations in the number of produced particles about the average.  $(\delta\phi)^2$  denotes a time-average)

$(1/\phi_c) \cong 0.6 \times 10^{-40} \text{sec}$ , which we use as the “starting” time (after inflation) in our calculation of the energy density<sup>4</sup>. With reference to the calculated potential in Fig. 1 ( $\phi_c = 10^{-3} M_P$ ,  $m_\phi^2 \cong (5 \times 10^{-10} M_P)^2$ ), and using (5a) with  $\epsilon \cong 10^{-5}$ , gives a matter energy density<sup>5</sup> for massive, cold inflatons at  $t_c$  of

$$\begin{aligned} \rho_M^\phi(t_c) &\cong \frac{1}{2} m_\phi^2 (\delta\phi)^2 \\ &\cong \frac{1}{2} (5 \times 10^{-10} M_P)^2 (10^{-11} M_P^2) \\ &\cong 2.5 \times 10^{46} \text{ GeV}^4 \end{aligned} \quad (7)$$

Use of (5b), with  $\epsilon = 10^{-7}$ , gives  $\rho_M^\phi(t_c) \sim 10^{46} \text{ GeV}^4$ . We assume the inflaton to be absolutely stable, or, as we shall show below, stable up to time scales much greater than the present age of the universe; for the latter we use in our calculations the approximate value  $t_0 \cong 4 \times 10^{17} \text{sec}$ . Note that the masses of the fermion  $L$  and the boson  $b$ , to which  $\phi$  is coupled by the interactions in (1), are approximately given by

$$\begin{aligned} m_L &\cong g(\phi_c)\phi_c \sim (10^{-2})(10^{16} \text{ GeV}) \\ &\cong 10^{14} \text{ GeV} \\ m_b &\cong \sqrt{\lambda_b(\phi_c)\phi_c} \sim \sqrt{10^{-4}}(10^{16} \text{ GeV}) \\ &\cong 10^{14} \text{ GeV} \end{aligned} \quad (8)$$

Both masses are much greater than  $m_\phi$ . We estimate the evolution of the matter energy density in (7) from  $t_c$  to  $t_0$ , using three distinct intervals. First, a relatively brief time interval up to  $t_H \cong 0.7 \times 10^{-34} \text{sec}$ , in which we assume that the universe is matter dominated. Here  $t_H$  is a time (calculated in Sect. 3 below), near which a massive ( $m_H \cong 10^{14} \text{ GeV}$ ), spin-zero boson  $H$ , decays into two photons, giving rise to a radiation energy density of about  $m_H^4 \cong 10^{56} \text{ GeV}^4$ . We then consider a second interval in which radiation dominates. This interval lasts until the usual time [6] of transition back to matter dominance, for which we use in our calculations  $t_M \cong 10^{11} \text{sec}$ . The third interval is then from  $t_M$  to  $t_0$ . The matter energy density of inflatons  $\rho_M^\phi$ , is proportional to  $R^{-3}(t)$ ; the scale factor  $R(t)$  develops as  $t^{2/3}$  when matter dominates, and as  $t^{1/2}$  when radiation dominates. Using (7), the result for the matter energy density for massive, cold inflatons in the present universe is then

$$\begin{aligned} \rho_M^\phi(t_0) &\cong \rho_M^\phi(t_c) \\ &\times \left( \frac{0.6 \times 10^{-40}}{0.7 \times 10^{-34}} \right)^2 \left( \frac{0.7 \times 10^{-34}}{10^{11}} \right)^{\frac{3}{2}} \\ &\times \left( \frac{10^{11}}{4 \times 10^{17}} \right)^2 \\ &\cong (2.5 \times 10^{46} \text{ GeV}^4)(0.75 \times 10^{-93}) \\ &\cong 2 \times 10^{-47} \text{ GeV}^4 \end{aligned} \quad (9)$$

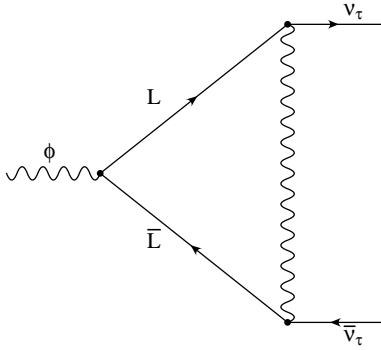
For the case of (5b) with  $\rho_M^\phi(t_c) \cong 10^{46} \text{ GeV}^4$ ,  $\rho_M^\phi(t_0) \cong 10^{-47} \text{ GeV}^4$ . These numbers are rather close to the closure energy density of about  $(2 - 3.5) \times 10^{-47} \text{ GeV}^4$ . A value  $\rho_M^\phi(t_0) \cong 1.5 \times 10^{-47} \text{ GeV}^4$  is calculated from the potential in Fig. 2 ( $\phi_c = 10^{-2} M_P$ ,  $m_\phi^2 \cong (4.25 \times 10^{-9} M_P)^2$ ), using (5a) with  $\epsilon \cong 10^{-7}$ . Regarding the energy scale  $\phi_c \cong 10^{16} \text{ GeV}$  (equivalently, the time<sup>4</sup>  $t_c \cong (1/\phi_c)$ ) at the end of inflation, it is noteworthy that a comparable scale has been obtained in a recent analysis [12] made to determine an upper-limit value for the Hubble parameter  $H(t)$ , at the end of inflation:  $(H_{\text{end}})_{\text{max}} \cong 4 \times 10^{-6} M_P$  (of the order of  $3 \times \phi^2/M_P$ ).

The result for  $\rho_M^\phi$  depends upon the size of the fluctuations<sup>4</sup> from (5,6). Therefore, we summarize the elements in recent calculations [8,9] which estimate<sup>6</sup> the parameter  $\epsilon$ . In [8],  $\epsilon$  is the product of two factors. One factor is a squared “initial” amplitude for oscillation, assumed to be  $\sim (10^{-1} M_P)^2$ , around an “initial” scale for  $\phi$ , which is assumed to be  $\phi \cong M_P$ . The second factor in effect reduces the final squared fluctuation because of the finite time for build-up; this factor depends explicitly upon  $\lambda$ , calculated as  $\sim (\ln(1/\lambda))^{-2} \cong 10^{-3}$ . Thus, the value of  $\epsilon$  is  $\sim (10^{-2})(10^{-3}) = 10^{-5}$  (in this case, the coefficient of the assumed “initial” value of  $\phi \cong M_P$  at the end of inflation). In [9], two factors are again involved; a smaller value of  $\epsilon \cong 10^{-7}$  is obtained for fluctuations in non-zero wave-number modes, largely because of a longer time interval for build up. The authors [9] have used some initial-condition fluctuations about which they only state: “the initial fluctuations are of the correct order of magnitude”, and that the parameter  $\lambda$  “regulates the magnitude of initial quantum fluctuations in non-zero (wave-number) modes relative to the magnitude of the zero mode”. The latter is implicitly assumed to be close to  $\phi = M_P$ . The essential physical point is that values of  $\epsilon$  in the range given in (6) can represent a maximum parameter<sup>4</sup> in the estimate of the fluctuation amplitude  $|\delta\phi|$ . On the other hand, our calculated value of  $\phi_c \cong 10^{-3} M_P$ , the classical field variable after inflation, is not as close to  $M_P$  as the values assumed and used, without calculation, in the papers [8,9] whose results we have summarized. From (5a,b), this is relevant for the size of  $(\delta\phi)^2$ .

<sup>4</sup> There is a time interval between  $t_c$  and the full development of the fluctuations (note [9]). Taking  $t_c$  as the approximate “starting” time for evolution of the full energy density gives a lower bound on the later densities, for a given initial density fixed by the specific choice of parameter  $\epsilon$ . In other words, increasing the effective  $t_c$  to  $t'_c$  allows for a decrease in  $\epsilon$  as  $(t_c/t'_c)^2$ ; this means that only (possibly much) smaller fluctuations are necessary, for the results in this paper. For example, consider  $t'_c \sim 10^{-36} \text{sec}$ . With  $(\delta\phi)^2 \sim m_\phi^2 \sim 10^{-13} \phi_c^2$  one has  $\rho_M^\phi(t'_c) \sim 2.5 \times 10^{38} \text{ GeV}^4$ , and  $\rho_M^\phi(t_0) \sim 5 \times 10^{-47} \text{ GeV}^4$  evolves from  $t'_c$ , as in (9) below

<sup>5</sup> Note that  $\lambda(\delta\phi)^2 < m_\phi^2$

<sup>6</sup> In [8], the estimate has a large element of assumptions concerning initial conditions. In [9], the detailed numerical simulation of the evolving system also rests upon a number of assumptions about initial conditions



**Fig. 3.** The process which gives rise to the eventual decay of massive quanta of the inflaton field, via a virtual  $L, \bar{L}$  pair, if  $m_{\nu_\tau} \neq 0$

### 3 Decay of the inflaton

A neutral lepton  $L$  (neutrino-like) and its antiparticle  $\bar{L}$ , play a role in virtual intermediate states in the dynamical model [4] for calculating the radiatively-corrected, tree-level potential for the inflaton field (Fig.1 and Fig.2). The lepton  $L$  is massive, from (8)  $m_L \cong 10^{14}$  GeV, as determined by the calculated values for  $\phi_c$  and for  $g(\phi_c)$ .

There is an interesting way in which massive  $L$  and  $\bar{L}$ , essentially at rest, can disappear: this is via a decay process caused by a minuscule mixing with a light neutrino, assuming that the latter has mass, say  $m_{\nu_\tau} \sim 1.8$  eV. This possible process has special interest because it can lead to the eventual decay of the inflaton. In Sect. 4, we show that the decay of  $L$  and  $\bar{L}$  also has possible relevance for the generation of a matter-antimatter asymmetry, specifically a neutrino-antineutrino-asymmetry which can occur at a time just somewhat prior to the electroweak symmetry-breaking time,  $\sim 10^{-12}$  sec. Consider the decay

$$L(\bar{L}) \rightarrow \nu_\tau(\bar{\nu}_\tau)H \quad (10)$$

We assume that the decay occurs because of a mixing interaction of the specific strength [13]<sup>7</sup>

$$\left(\sqrt{\frac{m_{\nu_\tau}}{m_L}}\right) (\bar{\psi}_{\nu_\tau}\psi_L H + \bar{\psi}_L\psi_{\nu_\tau} H)$$

The spin-zero boson  $H$  is Higgs-like (or  $\sigma$ -like) in that its coupling to fermions contains a mass factor (in effect  $\sqrt{m_{\nu_\tau} m_L}$ , relative to a scale  $F_H \sim m_H \sim m_L$ ). In general, with a coupling which mixes neutrino mass states, it is a distinct boson with a mass  $m_H = r m_L$ ,  $r \lesssim 1$ , i. e. of the order of  $10^{14}$  GeV. The decay width is

$$\begin{aligned} \Gamma(L(\bar{L}) \rightarrow \nu_\tau(\bar{\nu}_\tau)H) &= \frac{(1-r^2)^2}{8\pi} m_{\nu_\tau} \\ &\sim 10^{14} \text{ sec}^{-1} \end{aligned} \quad (11)$$

<sup>7</sup> The hypothesis of this kind of coupling between light neutrino mass states [13], is being tested by the CHOOZ reactor experiment [14], starting with  $\bar{\nu}_e$ . It can accommodate  $m_{\nu_\mu} \lesssim 0.03$  eV (for  $m_{\nu_\tau} \sim 1$  eV) and sizable  $\nu_e - \nu_\mu$  mixing. (Note that the alternative  $m_{\nu_\mu} \sim m_{\nu_\tau}$  with large mixing, can be accommodated)

where we have used  $m_{\nu_\tau} \cong 1.8$  eV, and  $(1-r^2) \sim 1$ . Because of the specific strength of the coupling, this decay width is *independent* of  $m_L$ , depending instead upon the assumed non-zero value of  $m_{\nu_\tau}$ . The lifetime is  $\tau_L \cong 10^{-14}$  sec, a number that is notable because it appears naturally close to the time of electroweak symmetry breaking,  $\sim 10^{-12}$  sec.

Now, *independently* of whether or not there is an “initial” energy density of  $L$  and  $\bar{L}$  at  $t_c$ , the inflaton will, in general, decay via a virtual intermediate state of an  $L\bar{L}$  pair:  $\phi \rightarrow \nu_\tau \bar{\nu}_\tau$ . The Feynman graph is shown in Fig.3. Calculation of the decay width gives,

$$\begin{aligned} \Gamma(\phi \rightarrow \nu_\tau \bar{\nu}_\tau) &= g^2 \left(\frac{m_{\nu_\tau}}{m_L}\right)^2 \left(\frac{m_\phi}{8\pi}\right) |\mathcal{M}|^2 \\ &\cong \left(\frac{m_{\nu_\tau}}{\phi_c}\right)^2 \left(\frac{m_\phi}{8\pi}\right) |\mathcal{M}|^2 \end{aligned}$$

where

$$|\mathcal{M}| \cong \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2 + m_L^2}{m_L^2}\right) \sim 10^{-2} \quad (12)$$

In (12), we have used  $m_L \cong g\phi_c$ ; thus  $\Gamma_\phi$  is *independent* of  $g$ .  $\Lambda$  which is taken as  $\sim 2m_L$  for an approximate estimate, is a cut-off on the intermediate-state momentum integral. Because of the minuscule number  $(m_{\nu_\tau}/\phi_c)^2 \cong 3.2 \times 10^{-50}$ , the lifetime for inflaton decay is,

$$\begin{aligned} \tau_\phi = \Gamma_\phi^{-1} &\cong \left(\frac{\phi_c}{m_{\nu_\tau}}\right)^2 (3.3 \times 10^{-29} \text{ sec}) \\ &\cong 10^{21} \text{ sec} \end{aligned} \quad (13)$$

Thus, we have used the calculated value of  $\phi_c \cong 10^{16}$  GeV for the potential in Fig.1, to estimate a possible inflaton lifetime about three orders of magnitude greater<sup>8</sup> than the present age of the universe. Under the assumption that  $\phi$  decays in this way, one may make an anthropic type of argument to the effect that  $m_{\nu_\tau}$  should not be greater than about 60 eV, since  $\tau_\phi > t_0$  must hold. Using  $t_0 \sim 10^{18}$  sec, (13) gives  $(10^{21})(1.8/m_{\nu_\tau})^2 > 10^{18}$ , and therefore  $m_{\nu_\tau} < 60$  eV. It is worth noting the closeness of this number to the Cowsik-McClelland bound [15] for the mass of a stable, light neutrino species,  $m_\nu < 91.5$  eV, (obtained by taking the reduced Hubble parameter  $h(t_0) < 1$ ).

Up to this point, we have not invoked any assumption concerning whether or not there is a significant “initial” energy density (at  $t \gtrsim t_c$ ) for  $L$  and  $\bar{L}$  (and/or for  $b$  quanta). We discuss this question in Sect. 4, which concerns a neutrino-antineutrino asymmetry. The  $H$  boson invoked for the decay in (10), exhibits a coupling to a light neutral lepton; therefore it could naturally be considered to decay promptly into two photons (presumably via a characteristic triangle-graph anomaly [16,17] involving charged particles). To calculate the lifetime, we use the

<sup>8</sup> For spin-zero  $\phi$ , one can make the lifetime much larger by applying  $(1+b\gamma_5)$  to  $\psi_{\nu_\tau}$  in the mixing interaction (after (10)), and letting  $b$  approach unity. This can be a reason for the projection, implying parity nonconservation

interaction  $(e^2/f_H)(\epsilon_{\mu\nu\sigma\rho}F_1^{\mu\nu}F_2^{\sigma\rho})H$ , where  $F_{1,2}^{\mu\nu}$  are the field tensors for the two photons,  $e^2$  is the squared electric charge, and the decay constant is a parameter, which we take as  $f_H \sim F_H \sim m_H \sim 10^{14}$  GeV. The lifetime for the  $H$  quantum is then

$$\begin{aligned} \tau_H &\cong (\Gamma(H \rightarrow 2\gamma))^{-1} = \left( \left( \frac{e^2}{4\pi} \right)^2 \left( \frac{\pi}{2} m_H \right) \right)^{-1} \\ &\cong 0.7 \times 10^{-34} \text{ sec} \end{aligned} \quad (14)$$

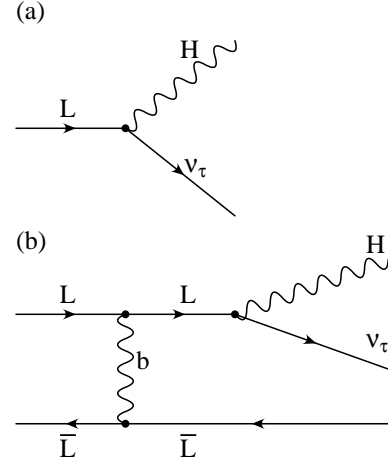
This is the time  $t_H$  used in (9) for the end of the first time interval in which matter dominates, and the beginning of the second time interval in which radiation dominates. For example, considering only photons, an energy density  $\rho_M^H \sim m_H^4 \sim 10^{56}$  GeV<sup>4</sup>, assumed to be present<sup>9</sup> at about  $t_H \cong 10^{-34}$  sec, and the decay  $H \rightarrow 2\gamma$ , leads to a radiation energy density which evolves to a present-day value of

$$\begin{aligned} \rho_\gamma(t_0) &\sim \rho_M^H(t_H) \left( \frac{10^{-34}}{10^{11}} \right)^2 \left( \frac{10^{11}}{4 \times 10^{17}} \right)^{8/3} \\ &\cong (10^{56} \text{ GeV}^4)(0.25 \times 10^{-107}) \\ &\cong 0.25 \times 10^{-51} \text{ GeV}^4 \end{aligned} \quad (15)$$

A relevant point about the above  $\rho_M^H(t_H)$  (as well as about other possible<sup>10</sup> (cold) matter energy densities such as  $\rho_M^{L,\bar{L}}(t_H)$ ,  $\rho_M^b(t_H)$ ), is that the values lie well below the maximum total energy density at the end of inflation which has been estimated [12] from the upper-limit value for the Hubble parameter at the end of inflation. This energy density is [12]  $\sim \frac{1}{8}((H_{\text{end}})_{\text{max}} M_P)^2 \sim 4 \times 10^{64}$  GeV<sup>4</sup>. Another point is that massive  $\phi$  quanta, and possible  $L$ ,  $\bar{L}$  quanta, have virtually no interaction contact with surrounding thermal conditions in the model of (1), from which the potential in Fig. 1 is calculated.

<sup>9</sup> As a means of initiating radiation, after inflation, this is more direct than invoking, under the assumption of some grand unification, a large number of hypothetical, massive particles, produced in hypothetical non-thermal conditions, and disappearing into radiation in such conditions, via a series of hypothetical decays. In any case, the maximal ‘‘thermal’’ condition defined [9] by an effective temperature of  $T_{\text{eff}} = \sqrt{12(\delta\phi)^2}$  is  $\sim 10^{14}$  GeV according to (7). Note that  $m_H \cong m_b \cong m_L$ , with  $H \rightarrow 2\gamma$ , produces the same energy scale for radiation, since  $m_b \cong m_L \cong 10^{14}$  GeV is calculated in (8)

<sup>10</sup> This estimate comes from consideration of the mixing of CP odd and even states in the  $(K_L^0 - K_S^0)$ -system, where this mixing is viewed as a remnant of the breakdown of discrete symmetries at a cosmological energy scale. (In [19], note (13) with  $M_P$  replaced by  $m_L$ . The small ratio of squared energy scales appears as  $(F_b^2/M_P^2)$  because  $m_L$  was assumed to be close to  $M_P$ , which is not the case in the present work. The primary time interval used in [19] is set by the Hubble parameter  $(H(m_L))^{-1}$  i. e. by the energy scale  $m_L$ .) Note that with  $F_b \ll \phi_c$ , corrections to the masses estimated in the present paper are small.  $\Gamma(b \rightarrow \phi\phi) \propto F_b^2 \cong (3.5 \times 10^{-23} \text{ sec})^{-1}$ . If  $m_b \gtrsim 2.2m_L$ ,  $\Gamma(b \rightarrow L\bar{L}) \gtrsim (1.5 \times 10^{-33} \text{ sec})^{-1}$  is much larger. This can be the origin of  $L, \bar{L}$ . For example, with  $\rho_M^b(t \sim 1/F_b \sim 2 \times 10^{-35} \text{ sec}) \sim 10^{46}$  GeV<sup>4</sup>



**Fig. 4.** **a** The decay of an isolated  $L$  (or  $\bar{L}$ ), via a mixing interaction to a light neutrino with mass  $m_{\nu_\tau}$ . **b** The decay of an  $L$  considered to be in a quasi-bound, metastable state with an  $\bar{L}$ , which condition is brought about by interaction involving the exchange of  $b$ -quanta. The process is not symmetric for  $L$  decay and  $\bar{L}$  decay

#### 4 CP noninvariance and a neutrino-antineutrino asymmetry

A neutral, spin-zero boson  $b$  plays an essential role in the dynamical model [4] for calculating the radiatively-corrected, tree-level potential for the inflaton field. We consider  $b$  to be a pseudoscalar (see appendix); from (8), it is massive,  $m_b \cong 10^{14}$  GeV, as determined by  $\lambda_b(\phi_c)$  and  $\phi_c$ . Such a pseudoscalar boson can play an important role in the early universe, where it can be the cause of the CP noninvariance that is necessary to arrive at a matter-antimatter asymmetry. CP noninvariance occurs spontaneously [18] if the  $b$ -field acquires a non-zero vacuum expectation value,  $F_b$ . In the following discussion, we shall assume that CP violation occurs, at least partly, through a spontaneous breakdown at a cosmological energy scale, (we have previously estimated [19] this to be at about  $F_b \sim 4 \times 10^{10}$  GeV). Since we have shown that the decay processes  $L(\bar{L}) \rightarrow \nu_\tau(\bar{\nu}_\tau)H$  occur at a time just somewhat prior to the time of electroweak symmetry breaking, they represent natural processes for generating an asymmetry in the number densities for neutrinos and antineutrinos. Such an asymmetry can result in a baryon-antibaryon asymmetry via processes which occur in the course of electroweak symmetry breaking [20, 21].

We have examined a novel mechanism for obtaining the asymmetry, one which relies explicitly on the spontaneous breakdown of CP invariance at a cosmological energy scale. The description of the process given here is intended only as a semi-quantitative *example*, one which is based upon the particle content and the interactions that occur in the dynamical model which produced the radiatively-corrected, tree-level potential in Fig. 1, and which allows for eventual decay of the massive, cold quanta of the inflaton field. The unusual aspect of the decay process is illustrated by the Feynman graphs in Fig. 4a,b. In (a) the decay process occurs for an isolated  $L$ ; in (b) the

process is different, the decay occurs for an  $L$  which we consider to be in a quasi-bound, metastable state with an  $\bar{L}$ . The quasi-bound state involves the exchange of  $b$ -quanta i. e. there is an “initial-state” interaction which gives rise to complex “scattering” amplitudes that multiply the decay vertex. There are, in fact, two different such amplitudes, corresponding to  $\gamma_5$  and 1 at the upper  $b$ -vertex in Fig. 4b. The spontaneous CP violation generates [18] a CP-violating, quantum interaction like  $(L\bar{L})b$  which has a scalar interaction character, starting from the CP-invariant, pseudoscalar interaction  $ig_b(\bar{L}\gamma_5 L)b$ . The scalar interaction is essential to have a non-zero lower vertex in Fig. 4b. Together with the decay vertex, of the general form  $(a(\bar{L}\nu_\tau + \bar{\nu}_\tau L) + b(\bar{L}\gamma_5\nu_\tau - \bar{\nu}_\tau\gamma_5 L))H$  (this does *not* violate CP), all of the elements are present that are necessary for obtaining a CP-violating difference between the partial rates for the particle and the antiparticle decay processes, as they occur in the quasi-bound configurations in Fig. 4b,  $L(+\bar{L}) \rightarrow \nu_\tau H(+\bar{L})$ ,  $\bar{L}(+L) \rightarrow \bar{\nu}_\tau H(+L)$ . The asymmetry in neutrino number density is proportional to the difference in the branching fractions for the process with particle and the process with antiparticle. Since there are initially equal numbers of  $L$  and  $\bar{L}$ , the asymmetry vanishes if the branching fractions are unity. There must be at least two processes by which  $L$  and  $\bar{L}$  “disappear”. We give the result of an estimate for the number-density asymmetry,

$$\begin{aligned} |A(n_{\nu_\tau} - n_{\bar{\nu}_\tau})| &\cong \left\{ \frac{\Gamma(L(+\bar{L}) \rightarrow \nu_\tau H(+\bar{L}))}{\Gamma(L \rightarrow \nu_\tau H)} \right\} \\ &\times \left\{ \frac{|\Gamma(L(+\bar{L}) \rightarrow \nu_\tau H(+\bar{L})) - \Gamma(\bar{L}(+L) \rightarrow \bar{\nu}_\tau H(+L))|}{\Gamma(L(+\bar{L}) \rightarrow \nu_\tau H(+\bar{L}))} \right\} \\ &\sim (\text{b. r.}) \left\{ \frac{g_b^2 \langle v^2 \rangle^{3/2}}{4\pi} \right\} \end{aligned} \quad (16)$$

The first factor in (16) is essentially the branching fraction for the decay process to occur from the quasi-bound state configuration;  $\langle v^2 \rangle^{1/2}$  represents a characteristic *internal* velocity for  $L(\bar{L})$  in the quasi-bound state configuration; and  $g_b^2 \cong 10^{-3}$  has been calculated (for the potential in Fig. 1). The factor in curly brackets in (16) is the result of the “initial-state” interaction. As an example, with (b. r.)  $\sim 0.1$  and  $\langle v^2 \rangle^{1/2} \lesssim 1$ , the number  $\lesssim 10^{-5}$  from (16) results in a small overall neutrino-antineutrino asymmetry. This is because the overall asymmetry must depend upon the number density of massive  $L(\bar{L})$  that are present at the decay time, relative to the number density of quanta in the radiation field at this time; this small ratio dilutes the asymmetry. Also, the CP-violating, scalar interaction is probably depressed by a factor like  $(F_b/m_L)$  (see appendix). Nevertheless, the relatively high energy involved in the neutrino-antineutrino asymmetry might allow an enhanced quark-antiquark number asymmetry at lower energy, essentially compensating the above-mentioned dilution factor.

The appearance of at least some part of a quark-antiquark asymmetry driven by an asymmetry between neutrinos and antineutrinos of relatively high energy, at the time of electroweak symmetry breaking, suggests an un-

usual possibility. The appearance of some baryons formed from quarks may then occur at a time after the time of photon decoupling, thus providing an impulse to structure formation.

## 5 Vacuum energy after inflation

There is a particular vacuum energy density at  $\phi_c$  (at time  $\sim t_c$ ),  $V_c(\phi_c) = \lambda(\phi_c)\phi_c^4 \sim -2 \times 10^{50} \text{ GeV}^4$ . The magnitude is only similar to the matter energy densities of massive, cold quanta which are present in the first time interval after inflation that we have considered in this paper,  $t_c \leq t \leq t_H$ . Consider a time-variation of  $|V_c(\phi_c)|$ . If  $|V_c(\phi_c)|$  were to fall like radiation, i. e. proportional to  $R^{-4}(t)$ , it would fall by about four orders of magnitude relative to the matter energy densities in the first interval. The assumption of matter dominance due to massive, cold  $\phi$ , and  $H$ , quanta in the first time interval, prior to  $H \rightarrow 2\gamma$ , is made. If this fall were to continue, the magnitude of the vacuum energy density comes to minuscule values,  $\sim 10^{-34} \text{ GeV}^4$  at the time of nucleosynthesis  $\sim 1$  sec, and  $\sim 10^{-73} \text{ GeV}^4$  at  $t_0$ . If  $|V_c(\phi_c)|$  were to fall like a matter energy density from about  $10^{-34}$  sec to  $\sim 1$  sec, a still relatively small value of  $\sim 10^{-17} \text{ GeV}^4$  occurs at the nucleosynthesis time. If continued to  $t_0$ ,  $\sim 2 \times 10^{-47} \text{ GeV}^4$  occurs; this is in itself interesting, being in magnitude similar to  $\rho_M(t_0)$  in (9).

A speculation involves the possible transfer of this energy to binding in an  $L\bar{L}$  “condensate”, which decays prior to the time of electroweak symmetry breaking, as discussed in Sect. 4. An estimate indicates that this binding could arise from the particular force which involves the trilinear couplings proportional to  $(4\lambda\phi_c)$  in (A8) of the appendix.

Alternatively, of course, the vacuum energy density at the position of the potential minimum can be adjusted to zero [4]. Equivalently, this negative energy density can be cancelled in a brief time by another positive density, such as that associated with the related field  $b$  (see appendix). Note that  $\lambda_b\phi_c^2 F_b^2$  is  $\sim 2 \times 10^{49} \text{ GeV}^4$ , similar to  $|V_c(\phi_c)|$  above.

Of the above possibilities, perhaps participation in  $L\bar{L}$  binding is particularly intriguing, because  $|V_c(\phi_c)|$  is in effect eliminated.

The discussion in this section leaves open the possibility that a positive residual vacuum energy density which decreases with increasing time can contribute to a relevant effective cosmological constant in the present universe. It is perhaps noteworthy that a vacuum energy density of about  $+2 \times 10^{50} \text{ GeV}^4$  at  $\sim 10^{-34}$  sec, which decreases as matter to the time of  $(L - \bar{L})$  decay  $\sim 10^{-14}$  sec and as radiation thereafter, attains a value of  $\sim 5 \times 10^{-48} \text{ GeV}^4$  at present.

## 6 Conclusions

Models for the period in the universe following a hypothetical period of inflation, which are based upon parti-

cle physics, tend to invoke somewhat arbitrary interaction forms and coupling strengths. In addition, there are a number of constraints imposed upon the dynamics in order to obtain some desired effects. Practically all of the hypotheses which are invoked cannot be tested (note footnote F9).

There are also primary questions: just how does the dynamics of a scalar field set up inflation, and what is the further role of this scalar field in the evolution of the universe?

In this paper, we have motivated our considerations by explicit, detailed calculation of the radiatively-corrected, tree-level potential which sets up inflation [4]. This potential is exhibited in Fig. 1, with its calculated maximum where inflation begins and calculated minimum where inflation ends. The calculation is carried out within a dynamical model with only a few essential fields, that is in addition to the inflaton field  $\phi$ , a massive, neutral lepton  $L$  and anti-lepton  $\bar{L}$ , and a massive, spin-zero boson  $b$ . The masses are calculated approximately from the potential and coupling parameters. A phenomenological consideration of the evolution of the matter energy density indicates the possibility that quanta of the inflaton field constitute a significant part of cold, dark matter in the present universe. When we consider a possible characteristic of the mixing between  $L$  and a light neutral lepton like  $\nu_\tau$ , assumed to have a mass, the possibility arises of the eventual decay of the inflaton. This occurs via virtual  $L\bar{L}$ :  $\phi \rightarrow \nu_\tau \bar{\nu}_\tau$ , with a calculable lifetime which depends upon the ratio  $(\phi_c/m_{\nu_\tau})^2$ . There are tests of these ideas. One involves comparison of the dark-matter energy density that is actually required by measurements, with that obtained for inflatons in improved calculations. A second involves testing the hypothesis that the mixing of neutrino mass states occurs via a coupling proportional to the square root of the mass ratio. The neutrino oscillation experiments can eventually do this, within the system of light neutrinos<sup>7</sup>. There, the hypothetical pattern involves one sizable mixing, either between  $\nu_e$  and  $\nu_\mu$ , or between  $\nu_\mu$  and  $\nu_\tau$ ; cancellations [13] cause the other mixings to be much smaller. The determination of the mass of the heaviest of these, presumably  $m_{\nu_\tau}$ , is relevant for the question of hot, dark matter, as well as to inflaton decay. A third direction involves examination of the possibility that CP invariance breaks down spontaneously at a cosmological energy scale [19]. If this is the only source of CP noninvariance (i. e. the quark mixing matrix is real), then the mixing of CP odd and even states in the  $(K_L^0 - K_S^0)$  system must be a remnant [19] of this high-scale symmetry breaking. CP violation primarily involving leptons enters the hadronic sector much depressed, via leptonic intermediate states. The analogue for neutral leptons of the mixing in the neutral kaon system is  $\bar{\nu}_\mu \nu_e \rightarrow \nu_\mu \bar{\nu}_e$ .

The above ideas, which we have illustrated quantitatively in this paper, provide some new possibilities, perhaps worthy of further study.

## Appendix: $\phi$ , $b$ , and $L$ in a primordial chiral symmetry, broken spontaneously and explicitly

There is a ‘‘toy’’ Lagrangian model which contains a number of aspects of the dynamics that we have discussed in this paper. The Lagrangian illustrates the possibility of a primordial chiral symmetry involving a neutral scalar  $\phi$ , a neutral pseudoscalar  $b$ , and a neutral lepton  $L$ . The symmetry is broken both spontaneously and explicitly. The model is patterned after the  $\sigma$ -model of pion physics [22, 23]. The Lagrangian density is [24]

$$\begin{aligned} \mathcal{L} = & \frac{i}{2} (\bar{L}\gamma_\mu(\partial^\mu L) - (\partial^\mu \bar{L})\gamma_\mu L) \\ & + \frac{1}{2} ((\partial_\mu \phi)^2 + (\partial_\mu b)^2) \\ & - \frac{\mu^2}{2} (\phi^2 + b^2) - g\bar{L}L\phi - ig\bar{L}\gamma_5 Lb \\ & - \lambda (\phi^2 + b^2)^2 \end{aligned} \quad (\text{A1})$$

$\mathcal{L}$  is invariant under the chiral transformation (with infinitesimal, constant  $\beta$ )

$$\begin{aligned} L & \rightarrow L - \frac{i\beta}{2}\gamma_5 L & \bar{L} & \rightarrow \bar{L} - \frac{i\beta}{2}\bar{L}\gamma_5 \\ \phi & \rightarrow \phi - \beta b & b & \rightarrow b + \beta\phi \end{aligned} \quad (\text{A2})$$

As a consequence  $\partial^\mu A_\mu(x) = 0$ , where a conserved axial vector current is given by

$$\begin{aligned} A_\mu & = -\frac{\delta \mathcal{L}}{\delta(\partial_\mu \beta(x))} \\ & = -\bar{L}\gamma_\mu\gamma_5 L + ((\partial_\mu \phi)b - (\partial_\mu b)\phi) \end{aligned} \quad (\text{A3})$$

Now add to the Lagrangian a piece  $\mathcal{L}'(\phi, b)$  given by

$$\mathcal{L}'(\phi, b) = c_\phi \phi + c_b b \quad (\text{A4})$$

Then  $\partial^\mu A(x) \neq 0$ ; it becomes

$$\partial^\mu A_\mu = -\frac{\delta \mathcal{L}}{\delta \beta} = (c_\phi b - c_b \phi) \quad (\text{A5})$$

There are, in general, certain non-zero matrix elements of  $A_\mu$ , defined by

$$\begin{aligned} \sqrt{2p_0^b} \langle 0 | A_\mu(x) | b \rangle & = i(p_\mu^b) e^{-i(p_\mu^b)x^\mu} \phi_c \\ \sqrt{2p_0^\phi} \langle 0 | A_\mu(x) | \phi \rangle & = -i(p_\mu^\phi) e^{-i(p_\mu^\phi)x^\mu} F_b \end{aligned} \quad (\text{A6})$$

Differentiating, and using (A5) gives

$$\begin{aligned} c_\phi & = m_b^2 \phi_c \\ c_b & = m_\phi^2 F_b \end{aligned} \quad (\text{A7})$$

We rewrite  $\mathcal{L}(\phi, b)$  in terms of fluctuating fields, that is  $\phi \rightarrow (\phi_c + (\delta\phi))$  and  $b \rightarrow (F_b + (\delta b))$ , using the notation  $(\delta\phi) = \tilde{\phi}$  and  $(\delta b) = \tilde{b}$ . We include all terms in  $\phi_c$  and terms linear in  $F_b$  - these terms contain the essential points

that we wish to make – because of the hypothesis that  $F_b \ll \phi_c$  (note footnote F10).

$$\begin{aligned} \mathcal{L}(\tilde{\phi}, \tilde{b}) = & \frac{i}{2} (\bar{L}\gamma_\mu(\partial^\mu L) - (\partial^\mu \bar{L})\gamma_\mu L) - m_L \bar{L}L \\ & + \frac{1}{2} \left( (\partial_\mu \tilde{\phi})^2 - m_\phi^2 \tilde{\phi}^2 \right) \\ & + \frac{1}{2} \left( (\partial_\mu \tilde{b})^2 - m_b^2 \tilde{b}^2 \right) \\ & - g \bar{L}L \tilde{\phi} - i g \bar{L}\gamma_5 L \tilde{b} - \lambda \left( \tilde{\phi}^2 + \tilde{b}^2 \right)^2 \\ & - 4\lambda \phi_c \tilde{\phi} \left( \tilde{\phi}^2 + \tilde{b}^2 + 2F_b \tilde{b} \right) \\ & - 4\lambda F_b \tilde{b} \left( \tilde{\phi}^2 + \tilde{b}^2 \right) \end{aligned}$$

with

$$\begin{aligned} & + (c_\phi - m_b^2 \phi_c) \tilde{\phi} + (c_b - m_b^2 F_b) \tilde{b} \quad (\text{A8}) \\ & m_L = g \phi_c \\ & m_\phi^2 = (\mu^2 + 12\lambda \phi_c^2) \\ & m_b^2 = (\mu^2 + 4\lambda \phi_c^2) \quad (\text{A9}) \end{aligned}$$

$\mathcal{L}(\tilde{\phi}, \tilde{b})$  does not treat  $\tilde{\phi}$  and  $\tilde{b}$  symmetrically, because  $F_b \neq \phi_c$ . Setting the coefficients of the last two terms in  $\mathcal{L}(\tilde{\phi}, \tilde{b})$  to zero ( $\langle 0|\mathcal{L}|\tilde{b}\rangle = 0 = \langle 0|\mathcal{L}|\tilde{\phi}\rangle$ ), gives

$$\begin{aligned} c_\phi &= m_b^2 \phi_c, \quad \text{as in (A7)} \\ c_b &= m_b^2 F_b \quad (= m_b^2 F_b \text{ from (A7)}) \quad (\text{A10}) \end{aligned}$$

Consider the limiting situation in which  $\mathcal{L}'$  vanishes [23]. In the usual Goldstone mode [25],  $\phi_c \neq 0$ . As  $c_\phi$  approaches zero,  $m_b^2$  approaches zero, and  $\mu^2$  approaches  $-4\lambda \phi_c^2 < 0$ . Then  $m_\phi^2$  tends to  $8\lambda \phi_c^2 > 0$ . If  $F_b \neq 0$  and  $c_b$  approaches zero,  $m_b^2$  must approach zero. Then  $\lambda$  approaches zero, and so  $\mu^2$  approaches zero through negative values. In other words, one appears to be just inside the boundary for spontaneous symmetry breaking.

In the model of (1), the required value of  $\lambda$  is non-zero but is indeed a very small number,  $10^{-13}$ . A reason for  $\lambda$  being such a small number has not been present before [2, 6]. The coefficient of the  $\phi^2 b^2$  interaction,  $\lambda_b$ , is much larger than  $2\lambda$ . There is an explicit breaking of the symmetry. This results in a relatively large mass for  $m_b$ , when the radiatively-corrected, tree-level potential for  $\phi$  acquires a minimum at  $\phi = \phi_c$ .

A unitary transformation makes manifest [18] the CP violation in the interactions in  $\mathcal{L}(\tilde{\phi}, \tilde{b})$ :

$$L \rightarrow e^{-i\gamma_5 \frac{\alpha}{2}} L \quad \bar{L} \rightarrow \bar{L} e^{-i\gamma_5 \frac{\alpha}{2}} \quad (\text{A11})$$

with  $\tan \alpha = (gF_b/m_L)$ . This results in the following changed form for a part of  $\mathcal{L}(\tilde{\phi}, \tilde{b})$ ,

$$\begin{aligned} & \left( m_L \bar{L}L + i g \bar{L}\gamma_5 L \tilde{b} + g \bar{L}L \tilde{\phi} \right) \rightarrow \\ & \left( \tilde{m}_L \bar{L}L + g \left( i \bar{L}\gamma_5 L (\cos \alpha) + \bar{L}L (\sin \alpha) \right) \tilde{b} \right. \\ & \left. + g \left( \bar{L}L (\cos \alpha) - i \bar{L}\gamma_5 L (\sin \alpha) \right) \tilde{\phi} \right) \quad (\text{A12}) \end{aligned}$$

where  $\tilde{m}_L = \sqrt{m_L^2 + (gF_b)^2}$ . CP violation occurs for  $\tilde{\phi}$ , as well as for  $\tilde{b}$ . (Note also the CP-violating mixing term  $(-8\lambda \phi_c F_b) \tilde{\phi} \tilde{b}$ , in (A8).)

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